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TECHNICAL MEMORANDUM

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DESIGN OF GENERALISED CHEBYSHEV SUSPENDED SUBSTRATE STRIPLINE FILTERS

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S U M M A R Y

A method for designing suspended substrate stripline (SSS) lowpass and highpass microwave filters is presented. Such filters can be realised by using a generalised Chebyshev lowpass prototype. An extension of this work is the development of SSS diplexers and multiplexers. The procedure for the design of a SSS contiguous diplexer is given.

SSS filters are compact and light-weight. Furthermore they are not as lossy as microstrip and more conventional stripline microwave filters.



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## 1. INTRODUCTION

A method for designing suspended substrate stripline (SSS) lowpass and highpass microwave filters is presented. Whilst the design of such filters has been outlined in reference 6, this treatment is somewhat fragmented and vague. The aim of this paper is to present a comprehensive and systematic procedure for designing such filters. The configuration of SSS is illustrated in figure 1. The dielectric between the ground planes is air except for the substrate. The substrate maintains the printed stripline circuit in suspension midway between the two ground planes.

In SSS filters a minimum of dielectric is used. This has the effect of minimising dielectric losses in the passband and increasing the temperature stability of the filters. There are other positive features with this type of filter design. The length of individual elements in SSS circuits is greater than the length of equivalent elements in ordinary stripline circuits. Therefore at high frequencies SSS circuits do not suffer from compression of component size. External fine tuning capabilities can be incorporated into the design. This eliminates the necessity for circuit trimming to achieve the desired performance.

To design these filters, a generalised Chebyshev lowpass prototype filter will be used (see figure 2). For a generalised Chebyshev lowpass prototype, the variation in impedance values of the circuit elements is small. With an impedance variation which is less than 2.5:1, printed circuit realisation is made easier. When an elliptic lowpass prototype is used, the high impedance variation (up to 10:1) makes circuit realisation difficult. Like equivalent elliptic lowpass filters, Chebyshev lowpass filters possess high selectivity properties. The frequency response of a lowpass prototype will show  $N-1$  transmission zeros at  $\omega_0$  and one transmission zero at infinity where  $\omega_0$  is the resonant frequency of the filter network shown in figure 2. A typical response for the lowpass prototype is illustrated in figure 3.

The magnitude of the element values(ref.1) related to a lowpass prototype is influenced by:-

- (a) the number of elements  $N$  in the filter (where  $N$ =odd number),
- (b) the return loss in the passband, and
- (c) the insertion loss in the stopband.

The later part of this paper will describe the design of a contiguous SSS diplexer using a Chebyshev lowpass prototype. Chebyshev lowpass prototypes are ideal for diplexer design because they ensure that a high degree of isolation is achieved between the low and high pass channels.

Furthermore, by accounting for and removing discontinuities present within the low- and highpass designs, external tuning can be minimised. Fine tuning, though, may be necessary because fluctuations may occur in the dielectric constant and the thickness of the strip conductor and/or substrate.

## 2. GENERALISED CHEBYSHEV LOWPASS PROTOTYPE

A generalised Chebyshev lowpass prototype (see figure 2) is used to design a SSS filter. This prototype is equiripple in the passband and has  $N-1$  transmission zeros at a frequency  $\omega_0$  near the bandedge.

The frequency response for this type of network is illustrated in figure 3. Its insertion loss is described by

$$L = 1 + \epsilon^2 \cdot \cosh^2 \left[ (N-1) \cdot \cosh^{-1} \left[ \omega \left( \frac{\omega_o^2 - 1}{\omega^2 - \omega_o^2} \right)^{\frac{1}{2}} \right] + \cosh^{-1} \omega \right] \quad (1)$$

where N is an odd number equal to the degree of the network. The ripple value  $\epsilon$  is given by

$$\epsilon = (10^{RL/10} - 1)^{-\frac{1}{2}} \quad (2)$$

where RL is the minimum return loss level (dB) in the passband. The minimum insertion loss in the stopband,  $L_m$ , occurs at  $\omega_m$  and  $\omega_1$  is the bandedge frequency of the stopband. It should be noted that  $\omega_o$ ,  $\omega_m$  and  $\omega_1$  are all normalised frequencies. The frequency  $\omega_m$  is given by

$$\omega_m^2 = \omega_o^2 + (N-1) \cdot \omega_o \cdot (\omega_o^2 - 1)^{\frac{1}{2}} \quad (3)$$

For given values of N, RL and  $L_m$ , the frequency  $\omega_1$  can be computed by solving equation (1) iteratively.

### 3. FILTER REALISATION

In this section an appropriate lumped lowpass prototype is transformed into lowpass and highpass distributed prototypes. These distributed prototypes are realised in printed circuit form. Consequently, the physical dimensions of the individual circuit elements that constitute the printed circuit lowpass and highpass filters are presented.

#### 3.1 SSS lowpass filter

The lumped lowpass prototype shown in figure 2 is transformed into a distributed lowpass prototype by using Richard's Transformation(ref.2)

$$p \rightarrow \omega_o \cdot \tanh(a_L p) \quad (4)$$

where p is the lumped complex frequency variable and  $a_L$  is a constant. The admittance of each shunt section in a lumped lowpass prototype is given by

$$Y = j \frac{C\omega}{1 - LC\omega^2} \quad (5)$$

where  $p = j\omega$ .

At resonance,  $LC = \omega_o^{-2}$ , therefore equation (5) becomes

$$Y = \frac{p C}{1 + p^2 / \omega_o^2} \quad (6)$$

After applying Richard's Transformation, the distributed lowpass prototype admittance becomes

$$\begin{aligned} Y &= \frac{\omega_o C_2(R) \tanh(a_L p)}{1 + \tanh^2(a_L p)} \\ &= \frac{\omega_o C_2(R)}{2} \cdot \tanh(2a_L p) \end{aligned} \quad (7)$$

Equation (7) is equivalent to the general expression of admittance for a shunt open circuit stub

$$Y = j Y_o \cdot \tan\left(\frac{\omega \ell}{v}\right) \quad (8)$$

therefore when equations (7) and (8) are equated and the complex frequency variable  $p$  is replaced by  $j\omega$ , one obtains

$$\begin{aligned} j Y_o \tan\left(\frac{\omega \ell}{v}\right) &= \frac{\omega_o C_2(R)}{2} \cdot \tanh(j2a_L \omega) \\ &= j \frac{\omega_o C_2(R)}{2} \cdot \tan(2a_L \omega) \end{aligned}$$

Hence

$$Y_o \tan\left(\frac{\omega \ell}{v}\right) = \frac{\omega_o C_2(R)}{2} \cdot \tan(2a_L \omega) \quad (9)$$

Equation (9) indicates that each resonator of the distributed prototype can be realised by a capacitive and therefore shunt open circuit stub. The normalised characteristic impedance of the  $R^{\text{th}}$  stub will be

$$Z_o = \frac{2}{\omega_o \cdot C_2(R)} \quad (10)$$

Furthermore, by equating the arguments in equation (9), the constant  $a_L$  is defined as

$$a_L = \frac{\ell}{2v} \quad (11)$$

where  $\ell$  = length of the shunt open circuit stub and  
 $v$  = velocity of light in free space.

The length of each stub is half a wavelength at the centre stopband frequency  $f_{cL}$ . Thus,  $f_{cL}$  is equal to twice the frequency at which  $N-1$  transmission zeros occur, that is

$$f_{cL} = 2 \cdot f_o \quad (12)$$

therefore the length of each stub is given as

$$\ell = \frac{v}{4f_o} \quad (13)$$

To compute the length  $\ell$ , an expression for  $f_o$  must be found. From Richard's Transformation

$$\begin{aligned} j\omega &= \omega_o \cdot \tanh(a_L p) \\ &= j \omega_o \cdot \tan(a_L 2\pi f) \end{aligned}$$

or

$$\omega = \omega_o \cdot \tan(a_L 2\pi f) \quad (14)$$

From equation (14), one obtains

$$a_L \cdot 2\pi f = \tan^{-1} \left( \frac{\omega}{\omega_o} \right) \quad (15)$$

Using equations (11) and (13)

$$a_L = \frac{1}{8f_o} \quad (16)$$

When substituting equation (16) into (15), one obtains

$$\frac{\pi}{4} \cdot \frac{f}{f_o} = \tan^{-1} \left( \frac{\omega}{\omega_o} \right)$$

therefore

$$f_o = \frac{\pi}{4} \cdot \frac{1}{\tan^{-1} (\omega/\omega_o)} \cdot f \quad (17)$$

At the bandedge frequency,  $f_{bL}$ , the normalised frequency  $\omega$  is unity, therefore equation (17) becomes

$$f_o = \frac{\pi}{4} \cdot \frac{1}{\tan^{-1} \left( \frac{1}{\omega_o} \right)} \cdot f_{bL} \quad (18)$$

Now that  $f_o$  is defined, the physical length of each stub is given by

$$\begin{aligned} \ell &= \frac{v}{2f_{cL}} \\ &= \frac{v}{4f_o} \\ &= \frac{v}{\pi \cdot f_{bL}} \cdot \tan^{-1} \left( \frac{1}{\omega_o} \right) \end{aligned} \quad (19)$$

The characteristic impedance  $Z_o$  of a lossless strip transmission line of constant width is given by (ref.3)

$$Z_o \sqrt{\epsilon_R} = \frac{\eta}{C/\epsilon} \quad (20)$$

where  $\epsilon_R$  = the relative dielectric constant of the medium in which the wave travels

$\eta$  = the impedance of free space ( $\approx 376.7 \Omega/\text{unit square}$ )

and  $C/\epsilon$  = the ratio of the static capacitance per unit length between conductors to the permittivity of the dielectric medium.

For the stripline configuration shown in figure 4, the static capacitance per unit length between the centre strip conductor and the ground planes is given by



$$C = 2C_P + 4C_F \quad (21)$$

where  $C_P$  = parallel-plate capacitance from the top or bottom of the strip conductor to the nearest ground plane

and  $C_F$  = fringing capacitance from the edge of the strip conductor to the nearest ground plane.

The overall parallel-plate capacitance is given by

$$\frac{C_P}{\epsilon} = 2 \cdot \frac{W/b}{1 - t/b} \quad (22)$$

where  $W$  and  $t$  are the width and thickness of the strip conductor respectively. Therefore the ratio  $C/\epsilon$  can be defined as

$$\frac{C}{\epsilon} = \frac{4W}{b-t} + 4 \frac{C_F}{\epsilon} \quad (23)$$

When substituting equation (23) into equation (20), one obtains the relationship between the characteristic impedance and the width of the strip conductor

$$W = \frac{b-t}{4} \left( \frac{376.7}{Z_0 \sqrt{\epsilon_R}} - 4 \frac{C_F}{\epsilon} \right) \quad (24)$$

When the expression for  $Z_0$  in equation (10) is scaled for 50  $\Omega$  terminations, then substituted into equation (24), the physical width of each resonator is given by

$$W = \frac{b-t}{4} \cdot \left( \frac{3.767 \omega_0 C_2(R)}{\sqrt{\epsilon_R}} - 4 \frac{C_F}{\epsilon} \right) \quad (25)$$

For SSS the dielectric is almost entirely air therefore to a good approximation,  $\epsilon_R$  is unity. The fringing capacitance  $C_F/\epsilon$  can be determined from figure 5(ref.4).

The series lumped inductors are approximated by high impedance striplines. When the lumped network is transformed into a distributed network, the series inductors are transformed into series short-circuit stubs. The equivalent circuit for a length of transmission line is a  $\pi$ -network, as shown in figure 6. The shunt capacitors in the  $\pi$ -network relate to the parallel-plate capacitance between the central strip conductor and the ground planes.

The design dimensions for the printed circuit series line can be derived by equating the series impedance of the  $\pi$ -network to the impedance of the corresponding series short-circuit stub (at frequency  $f = f_{bL}$ ).

The series impedance of the  $\pi$ -network is

$$Z = jZ_L \sin\left(\frac{\omega \ell_L}{v}\right) \cong jZ_L \cdot \frac{2\pi f_{bL} \ell_L}{v} \quad (26)$$

where  $Z_L$  = normalised characteristic impedance of the series element  
and  $\ell_L$  = physical length of the inductive line.

The impedance of the short circuit stub is derived from Richard's Transformation and hence given as

$$Z = j L_r \omega_o \cdot \tan\left(\frac{\omega_{bL} \ell_{SSC}}{v}\right) \quad (27)$$

where  $L_r$  = prototype element value(ref.1) for the corresponding series element

and  $\ell_{SSC}$  = length of short-circuit stub which is a quarter of a wavelength at the centre stopband frequency  $f_{cL}$ .

When equations (26) and (27) are equated they generate

$$\ell_L = \frac{L_r \omega_o}{Z_L} \cdot \frac{v}{2\pi f_{bL}} \cdot \tan\left(\frac{\omega_{bL} \ell_{SSC}}{v}\right) \quad (28)$$

$Z_L$  must be evaluated prior to computing the length of each inductive line. If one sets  $\epsilon_R = 1.0$  and  $Z_o = 50.Z_L$  (for a filter that will work between 50  $\Omega$  terminations) in equation (24),  $Z_L$  is defined as

$$Z_L = \frac{7.534}{\frac{4W}{b-t} + \frac{4C_F}{\epsilon}} \quad (29)$$

### 3.2 SSS highpass filter

The lumped lowpass prototype is transformed into a highpass distributed prototype by applying the frequency transformation

$$p \rightarrow \frac{\omega_o}{\tanh(a_H p)} \quad (30)$$

where, as previously,  $p$  is the lumped complex frequency variable and  $a_H$  is a constant.

The admittance of each section in the lumped lowpass prototype has been given in equation (6). When applying the frequency transformation (30), the admittance for each shunt section in the highpass prototype becomes

$$\begin{aligned}
 Y &= \frac{\omega_o C_2(R) \cdot \tanh(a_H p)}{\tanh^2(a_H p) + 1} \\
 &= -\omega_o C_2(R) \cdot j \frac{\tan(a_H \omega)}{\tan^2(a_H \omega) - 1} \\
 &= j \frac{\omega_o C_2(R)}{2} \cdot \tan(2a_H \omega) \quad (31)
 \end{aligned}$$

When equation (31) is equated with the general expression for a shunt section, as given by equation (8), each shunt section of the highpass distributed prototype will be represented by a capacitive and therefore shunt open circuit stub. The normalised characteristic impedance of the  $R^{\text{th}}$  stub will be

$$Z_o = \frac{2}{\omega_o \cdot C_2(R)}$$

as is the case with the lowpass distributed prototype. Therefore, when derived from the same Chebyshev lowpass prototype, lowpass and highpass distributed prototypes have shunt sections with identical characteristic impedances.

When the arguments from equations (8) and (31) are equated, the constant  $a_H$  is defined as

$$\begin{aligned}
 a_H &= \frac{\ell}{2v} \\
 &= \frac{1}{8f_o} \quad (32)
 \end{aligned}$$

where the length  $\ell$  of these stubs is a quarter wavelength at  $f_o$ .  $f_o$  is the frequency at which  $(N-1)$  transmission zeros occur for the SSS highpass filter.

From the frequency transformation expression given by equation (30)

$$\begin{aligned} j\omega &= \frac{\omega_o}{\tanh(j a_H^2 \pi f)} \\ &= -j \frac{\omega_o}{\tan(a_H^2 \pi f)} \end{aligned}$$

or

$$\omega = \frac{\omega_o}{\tan(a_H^2 \pi f)} \quad (33)$$

Given that  $-\omega_o$  corresponds to  $\omega_o$ , equation (33) can be re-written as

$$a_H^2 \pi f = \tan^{-1} \left( \frac{\omega_o}{\omega} \right) \quad (34)$$

At the bandedge frequency,  $\omega=1$ . Consequently, when substituting equation (32) into equation (34) at the bandedge frequency,  $f_0$  is expressed as

$$f_0 = \frac{\pi}{4} \cdot \frac{1}{\tan^{-1} \omega_o} \cdot f_{bH} \quad (35)$$

where  $f_{bH}$  = bandedge frequency of the highpass filter.

Therefore the physical length of the shunt open circuit stubs is given as

$$\begin{aligned} \ell &= \frac{\lambda_o}{4} \\ &= \frac{v}{4f_o} \\ &= \frac{v}{\pi \cdot f_{bH}} \cdot \tan^{-1} \omega_o \end{aligned} \quad (36)$$

The frequency transformation (30) indicates that the series elements of the distributed highpass prototype are open circuit stubs a quarter of a

wavelength long at the centre passband frequency. These stubs should generate a zero impedance at the centre passband frequency.

The series elements are approximated by inhomogeneous broad-side coupled striplines in printed circuit form (see figure 7). The equivalent circuit for this inhomogeneous section is an open-circuit digital section(ref.5). The equivalent circuit is illustrated in figure 8. In the remaining part of this sub-section, design equations will be generated to compute the physical length and width of the broadside-coupled section.

The open-circuit digital section has the following ABCD parameters(ref.5);

$$A = \frac{Z_{oe} \cdot \cot\theta_e + Z_{oo} \cdot \cot\theta_o}{Z_{oe} \cdot \csc\theta_e - Z_{oo} \cdot \csc\theta_o} = D \quad (37)$$

$$B = \frac{j}{2} \cdot \frac{Z_{oe}^2 + Z_{oo}^2 - 2 \cdot Z_{oe} \cdot Z_{oo} (\cot\theta_e \cdot \cot\theta_o + \csc\theta_e \cdot \csc\theta_o)}{Z_{oe} \cdot \csc\theta_e - Z_{oo} \cdot \csc\theta_o} \quad (38)$$

$$C = j \frac{2}{Z_{oe} \cdot \csc\theta_e - Z_{oo} \cdot \csc\theta_o} \quad (39)$$

$Z_{oe}$  and  $Z_{oo}$  are the even and odd mode impedances for an inhomogeneous broadside-coupled section.  $Z_{oe}$  and  $Z_{oo}$  are related to the physical width  $W$  of each coupled section. The modal electrical lengths  $\theta_e$  and  $\theta_o$  are defined as

$$\theta_e = \frac{\omega \ell}{v_e}$$

and

$$\theta_o = \frac{\omega \ell}{v_o}$$

where  $v_e$  and  $v_o$  are the phase velocities for the even and odd modes respectively and  $\ell$  is the physical length of the inhomogeneous coupling section. Appendix I describes a method used to compute  $Z_{oe}$ ,  $Z_{oo}$ ,  $v_e$  and  $v_o$  from  $W$ .

In the design of SSS highpass filters, all inner series elements are realised by identical inhomogeneous sections. This approximation is possible because the lumped lowpass prototype element values are almost equal. The end series elements are realised by different inhomogeneous sections.

Furthermore, the inhomogeneous section corresponds to a  $\pi$ -network as shown in figure 9 (see reference 6).  $Z'_H$  and  $Y'_H$  are the series impedance and shunt admittance respectively of the  $\pi$ -network of the inner identical inhomogeneous sections.

When designing the inhomogeneous sections, two conditions must be fulfilled(ref.6). Firstly, at the bandedge frequency  $f_{bH}$ , the impedance of each inhomogeneous section is equal to the corresponding series impedance in the distributed highpass prototype. Hence

$$\left. \begin{aligned} \text{Im}(Z'_{in}) &= \text{Im}(Z_{in}) \end{aligned} \right\} \text{ at } f = f_{bH} \quad (40)$$

and

$$\left. \begin{aligned} Z'_H &= -j \frac{L_o(R) \omega_o}{\tan(a_H \omega)} \end{aligned} \right\} \quad (41)$$

where  $Z_{in}$  is the series impedance generated by the end series open circuit stubs and the corresponding generator or load resistance in the distributed highpass prototype, and  $Z'_{in}$  is the corresponding impedance produced by the end inhomogeneous sections in the final printed circuit filter.

Secondly, at the centre passband frequency  $f_{CH}$ , the impedance of the distributed highpass and inhomogeneous sections will resonate, hence

$$\left. \begin{aligned} \text{Im}(Z'_{in}) &= 0 \end{aligned} \right\} \text{ at } f = f_{CH} \quad (42)$$

and

$$\left. \begin{aligned} Z'_H &= 0 \end{aligned} \right\} \quad (43)$$

Simultaneously, at the centre passband frequency, the total shunt admittance produced by the inhomogeneous sections and the shunt resonators will be zero. For an N-section SSS highpass filter network, as illustrated in figure 10, the number of shunt sections is given by

$$M = \frac{N-1}{2}$$

For M even,  $R = N-M$ , and for M odd,  $R = N-M-1$  where R refers to the specific capacitive and inductive elements  $C_2(R)$  and  $L_0(R)$  of the Chebyshev lowpass prototype used in the design procedure.  $C_2(R)$  and  $L_0(R)$  appear in equations (31) and (41) respectively.

Given the admittance for each shunt resonator (31) and the structure of the network shown in figure 10, the total shunt admittance for  $N \geq 9$  is

$$j \frac{\omega_o C_2(R)}{2} \cdot \tan(4\pi a_H f) + 2Y_H'(f) = 0 \quad (44(a))$$

For  $N \leq 7$ , that is  $M = 2, 3$ , equation (44(a)) is changed to

$$j \frac{\omega_o \cdot C_2(R)}{2} \cdot \tan(4\pi a_H f) + Y_H'(f) + Y_H''(f) = 0 \quad (44(b))$$

The solution to either equations (44(a)) or (44(b)) will be  $f = f_{CH}$ . Therefore once  $f_{CH}$  has been computed, the design equations (40) to (43) can be solved.

The ABCD parameters for the inhomogeneous coupling section shown in figure 9 can be deduced from equations given in Appendix II. They are given by

$$A = 1 + Z_H' \cdot Y_H' = D \quad (45)$$

$$B = Z_H' \quad (46)$$

and

$$C = 2 \cdot Y_H' + Z_H' \cdot (Y_H')^2 \quad (47)$$

Since the inhomogeneous section is equivalent to an open circuit digital section, equation (46) can be equated with equation (38) to form

$$Z_H' = \frac{j}{2} \cdot \frac{Z_{oe}^2 + Z_{oo}^2 - 2Z_{oe} \cdot Z_{oo} (\cot\theta_e \cdot \cot\theta_o + \csc\theta_e \cdot \csc\theta_o)}{Z_{oe} \cdot \csc\theta_e - Z_{oo} \cdot \csc\theta_o} \quad (48)$$

Furthermore, when using equation (46),  $Z_H'$  in equation (45) can be replaced by B. Consequently,  $Y_H'$  is given by

$$Y'_H = \frac{A-1}{B} \quad (49)$$

$Y'_H$  can be expressed by components representing even and odd mode impedances and electrical lengths by replacing A and B by the expressions given by equations (37) and (38) respectively. Hence  $Y'_H$  becomes

$$Y'_H = \frac{2}{j} \cdot \frac{Z_{oe} (\cot\theta_e - \csc\theta_e) + Z_{oo} (\cot\theta_o + \csc\theta_o)}{Z_{oe}^2 + Z_{oo}^2 - 2 \cdot Z_{oe} \cdot Z_{oo} (\cot\theta_e \cdot \cot\theta_o + \csc\theta_e \cdot \csc\theta_o)} \quad (50)$$

The general expression for the input impedance  $Z'_{in}$ , which consists of ABCD parameters of the end inhomogeneous sections, is given by

$$\begin{aligned} Z'_{in} &= \frac{A Z_L + B}{C Z_L + D} \\ &= \frac{A + B}{C + D} \end{aligned} \quad (51)$$

where the load impedance  $Z_L$  is normalised to unity.

From the definitions given for the ABCD parameters by equations (37), (38) and (39), the quantity  $I_m(Z'_{in})$  is

$$\begin{aligned} \text{Im}(Z'_{in}) &= \frac{.5[Z_{oe}^2 + Z_{oo}^2 - 2 \cdot Z_{oe} \cdot Z_{oo} (\cot\theta_e \cdot \cot\theta_o + \csc\theta_e \cdot \csc\theta_o)] \cdot [Z_{oe} \cdot \cot\theta_e + Z_{oo} \cdot \cot\theta_o]}{[Z_{oe} \cdot \cot\theta_e + Z_{oo} \cdot \cot\theta_o]^2 + 4} \\ &\quad - \frac{2 \cdot [Z_{oe} \cdot \cot\theta_e + Z_{oo} \cdot \cot\theta_o]}{[Z_{oe} \cdot \cot\theta_e + Z_{oo} \cdot \cot\theta_o]^2 + 4} \end{aligned} \quad (52)$$

In equations (48), (50) and (52),  $Z_{oe}$  and  $Z_{oo}$  are normalised for 1  $\Omega$  terminations.

Finally, as mentioned earlier,  $Z_{in}$  is the impedance generated by the series connection of the load impedance and the end open circuit stub. Therefore



$$\begin{aligned}
 Z_{in} &= Z_L - j \frac{Z_o}{\tan\left(\frac{\omega \ell}{v}\right)} \\
 &= 1 - j \frac{L_o(1) \cdot \omega_o}{\tan\left(\frac{\omega \ell}{v}\right)}
 \end{aligned} \tag{53}$$

where  $Z_{in}$  is normalised for 1  $\Omega$  terminations.

Therefore

$$\text{Im}(Z_{in}) = - \frac{L_o(1) \cdot \omega_o}{\tan\left(\frac{\omega \ell}{v}\right)} \tag{54}$$

Hence given the definitions for  $Z'_H$ ,  $Y'_H$ ,  $\text{Im}(Z'_{in})$  and  $\text{Im}(Z_{in})$ , the design equations can be solved. The solution to equation (44) is used initially in conjunction with equations (40) and (42) to compute the physical width and length of the end inhomogeneous coupling sections. Similarly when the solution to equation (44) is used together with equations (41) and (43), the dimensions of the inner inhomogeneous sections are computed.

An efficient method of solving equations (40) to (44) is by way of suitable optimisation routines. To solve equation (44) one needs to find the values of the parameters  $W$ ,  $\ell$  and  $f$  that minimise the function

$$F = \begin{cases} \left[ j \frac{\omega_o \cdot C_2(R)}{2} \cdot \tan(4\pi a_H f) + 2 \cdot Y'_H(f) \right]^2 & \text{for } N \geq 9 \\ \left[ j \frac{\omega_o \cdot C_2(R)}{2} \cdot \tan(4\pi a_H f) + Y'_H(f) + Y''_H(f) \right]^2 & \text{for } N \leq 7 \end{cases} \tag{55}$$

The optimum value of the parameter  $f$  is equivalent to  $f_{CH}$ . The parameters  $W$  and  $\ell$  refer to the width and length respectively of the inhomogeneous coupling section.

The solution to equations (40) and (42) will determine the optimum width and length of the end inhomogeneous coupling section. To achieve this, the sum of squares(ref.7) of the following two functions

$$\begin{aligned}
 g_1 &= \text{Im}(Z'_{in}) - \text{Im}(Z_{in}) \\
 &= \text{Im}(Z'_{in}) + \frac{L_o(1) \cdot \omega_o}{\tan\left(\frac{\omega \ell}{v}\right)} \quad \text{at } f = f_{bH}
 \end{aligned} \tag{56}$$

and

$$g_2 = \text{Im}(Z'_{in}) \quad \text{at } f = f_{CH} \quad (57)$$

is minimised.

Hence the overall function being minimised is

$$G = g_1^2 + g_2^2 \quad (58)$$

The same procedure is used to solve equations (41) and (43). In this case the two functions involved are

$$g_1 = Z'_H + j \frac{L(R) \cdot \omega}{\tan\left(\frac{\omega \ell}{v}\right)} \quad \text{at } f = f_{bH} \quad (59)$$

and

$$g_2 = Z'_H \quad \text{at } f = f_{CH} \quad (60)$$

### 3.3 SSS diplexer

The final phase in SSS filter design constitutes the diplexing of individual low- and highpass networks. In SSS configuration, the two networks are connected in parallel as shown in figure 11.

To produce a good VSWR across the diplexer bandwidth, the input impedance of a diplexer should approximate the source impedance. The input admittance is given as

$$Y_T = Y_L + Y_H \quad (61)$$

where  $Y_L$  and  $Y_H$  are the input admittances of the lowpass and highpass networks respectively.

For normalised impedances, ideal impedance matching occurs when

$$\begin{aligned} \text{Re}(Y_T) &= \text{Re}(Y_L) + \text{Re}(Y_H) \\ &= 1 \end{aligned} \quad (62)$$

and

$$\begin{aligned} \text{Im}(Y_T) &= \text{Im}(Y_L) + \text{Im}(Y_H) \\ &= 0 \end{aligned} \quad (63)$$

When conditions (62) and (63) are satisfied, the Chebyshev low- and highpass networks are contiguously diplexed. In a contiguous diplexer, the low and highpass networks share a common cross-over frequency. The insertion loss at the cross-over frequency is 3 dB.

To achieve the ideal impedance matching conditions in a contiguous diplexer, the first series and first shunt elements of the constituent low- and highpass networks are modified. A lumped element diplexer can be regarded as a linear network where the highpass network is considered as a shunt element cascaded with the lowpass network as shown for the case  $N = 7$  in figure 12. The element values denoted by the asterisks in figure 12 refer to those which are modified by optimisation. These element values are related to the lowpass prototype values (see figure 2) by frequency and impedance scaling formulas. Furthermore, given that the component lowpass and highpass filters have identical cut-off frequencies, and their respective zero transmission frequencies are held constant, it can be shown that the elements  $C_{L2}$ ,  $C_{H1}$ ,  $L_{H2}$  and  $C_{H2}$  are dependent on either  $L_{L1}$  or  $L_{L2}$ :

$$C_{L2} = \frac{1}{4\pi^2 f_{oL}^2} \cdot \frac{1}{L_{L2}} \quad (64)$$

$$C_{H1} = \frac{1}{4\pi^2 f_{bL}^2} \cdot \frac{1}{L_{L1}} \quad (65)$$

$$L_{H2} = \left( \frac{f_{oL}}{f_{bL}} \right)^2 \cdot L_{L2} \quad (66)$$

and

$$C_{H2} = \frac{1}{4\pi^2 f_{bL}^2} \cdot \frac{1}{L_{L2}} \quad (67)$$

where  $f_{bL}$  and  $f_{oL}$  are the respective cut-off and zero transmission frequencies of the lowpass filter.

Therefore  $L_{L1}$  and  $L_{L2}$  are the only parameters that require optimisation. The modified values for the elements  $C_{L2}$ ,  $C_{H1}$ ,  $L_{H2}$  and  $C_{H2}$  are evaluated by substituting the optimum values of  $L_{L1}$  and  $L_{L2}$  into equations (64) to (67). The optimum values for  $L_{L1}$  and  $L_{L2}$  will minimise the function  $G$  which consists of the sum of squares(ref.7) of the functions  $g_i$ :

$$G = g_1^2 + g_2^2 + \dots + g_n^2$$

For a source impedance of 50  $\Omega$

$$g_i(L_{L1}, L_{L2}) = \text{Re}(Y_T) \Big|_{f=f_i} - 0.02 \quad (68)$$

where  $f_i$  are selected frequencies over the diplexer bandwidth. The expression for G can be rewritten as

$$G = \sum_{i=1}^n g_i^2(L_{L1}, L_{L2}) \quad (69)$$

Therefore from the optimum component values  $L'_{L1}$  and  $L'_{L2}$ , the new component values  $C'_{L2}$ ,  $C'_{H1}$ ,  $C'_{H2}$  and  $L'_{H2}$  are computed. These modified component values will generate an optimised lumped element contiguous diplexer.

Using equation (10), the modified normalised characteristic impedance  $Z_o$  of the first shunt section in the low- and highpass branches of the printed circuit diplexer is given by

$$Z_o = \frac{2}{C'_2(N) \cdot \omega_o} \quad (70)$$

$C'_2(N)$  is related to  $C'_{L2}$  by the combined impedance and frequency scaling formula

$$C'_2(N) = 2 \cdot \pi \cdot f_{bL} \cdot Z_L \cdot C'_{L2}$$

The final step in the design of a printed circuit diplexer involves optimising the lengths of the first inhomogeneous and inductive sections of the high- and lowpass channels respectively so that

$$\text{Re}(Y_T) = 0.02 \quad (\text{for } Z_G = 50 \Omega)$$

over the diplexer bandwidth.

Consequently the above printed circuit modifications will form a SSS diplexer with a good VSWR across its entire bandwidth.

## 4. THEORETICAL RESULTS

A generalised Chebyshev lowpass prototype filter of degree  $N=7$  with minimum stopband insertion loss and passband return loss of 50 dB and 20 dB respectively, has been used to design SSS lowpass, highpass and diplexer networks with 6 GHz bandedge frequencies and cross-over frequency respectively.

The printed circuit dimensions of each filter network have been computed using the mathematical relations outlined in Section 3. The printed circuit layout for the lowpass filter is shown in figure 13.

For the 6 GHz lowpass filter, the inductive lines have the physical lengths

$$\ell_{L1} = \ell_{L7} = 2.541 \text{ mm}$$

$$\ell_{L3} = \ell_{L5} = 5.216 \text{ mm}$$

and a common line width

$$W_L = 0.635 \text{ mm}$$

The shunt stubs have physical lengths of

$$\ell_{L2} = \ell_{L4} = \ell_{L6} = 8.895 \text{ mm}$$

and their widths are given as

$$W_{L2} = W_{L6} = 1.869 \text{ mm}$$

and

$$W_{L4} = 2.513 \text{ mm.}$$

The printed circuit layout for the highpass filter is illustrated in figure 14. For a 6 GHz highpass filter, the lengths and widths of the shunt stubs are

$$\ell_{H2} = \ell_{H4} = \ell_{H6} = 16.105 \text{ mm}$$

$$W_{H2} = W_{H6} = 1.869 \text{ mm}$$

and

$$W_{H4} = 2.513 \text{ mm}$$

respectively.

The lengths and widths of the series broadside-coupled sections are

$$\ell_{H1} = \ell_{H7} = 3.226 \text{ mm}$$

$$\ell_{H3} = \ell_{H5} = 3.380 \text{ mm}$$

and

$$W_{H1} = W_{H7} = 0.605 \text{ mm}$$

$$W_{H3} = W_{H5} = 0.264 \text{ mm}.$$

When the lowpass and highpass networks are diplexed, as described in figure 15, the following changes are made to the relevant constituent networks,

$$\ell_{L1} \rightarrow 4.390 \text{ mm}$$

$$W_{L2} \rightarrow 3.157 \text{ mm}$$

$$\ell_{H1} \rightarrow 2.441 \text{ mm}$$

and

$$W_{H2} \rightarrow 3.157 \text{ mm}.$$

Theoretical results for the above SSS filter networks are illustrated in figures 16, 17, 18, 19, 20 and 21.

## 5. CONCLUSION

The insertion loss responses, as presented in Section 4, for the SSS lowpass and highpass filters are not quite identical to the responses pertaining to the equivalent distributed lowpass and highpass networks. Response anomalies are apparent in their stopband performance. The response anomalies appear because approximations are made when realising the series elements of the SSS filters.

This aside, the performance of both filters is good. The SSS lowpass filter has a stopband which is at least an octave wide and a maximum VSWR of 1.35 over a major portion of the passband. The SSS highpass filter exhibits very good stopband performance from dc to the cut-off frequency and its passband is up-to an octave wide. These positive features make SSS lowpass and highpass filters ideal for diplexer applications. The performance of the 6 GHz SSS diplexer matches that of the equivalent distributed network diplexer.

When realising a SSS filter, the discontinuities present in the stripline structures must be removed or reduced. Discontinuities introduce unwanted parasitic reactances. Common types of discontinuities present in SSS filter networks include open ends, steps in width and T-junctions. If all parasitic reactances are eliminated, external tuning can be reduced to a minimum. Some external capacitive fine tuning of the central shunt resonators may be necessary to overcome performance anomalies that arise from fluctuations in the dielectric constant of the substrate and the thickness of the strip conductor and/or substrate. The above discontinuities are discussed in detail in Appendix III.

When using the optimisation routines for the design of highpass filters and diplexers, suitable starting parameter values should be chosen. This ensures that the final optimised parameter values generate a global minimum, instead of a localised minimum, for the function G.

SSS contiguous multiplexers consisting of highly selective SSS lowpass and highpass filter groups can be employed in the design of compact and light-weight ultra-wide band radar warning receivers.

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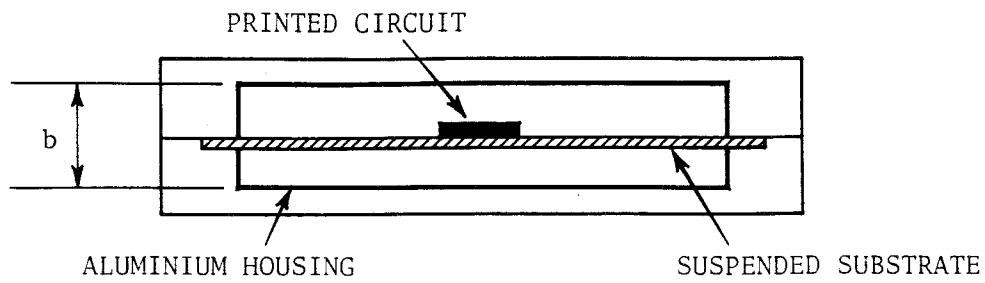


Figure 1. Configuration of suspended substrate stripline where  $b$  is the ground plane spacing

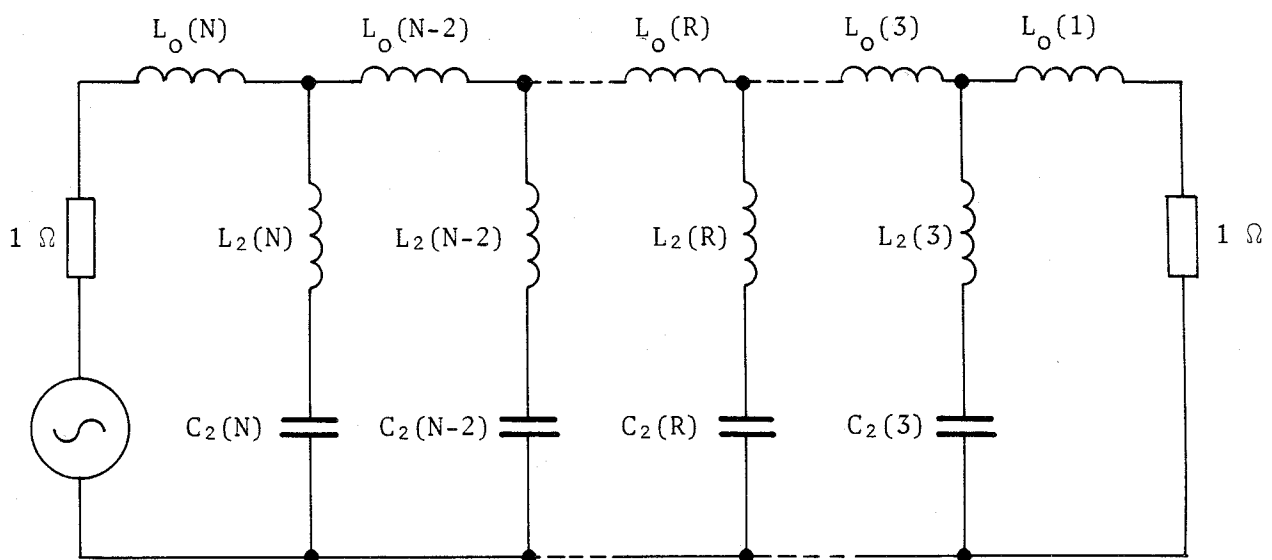


Figure 2. The generalised Chebyshev lowpass prototype filter

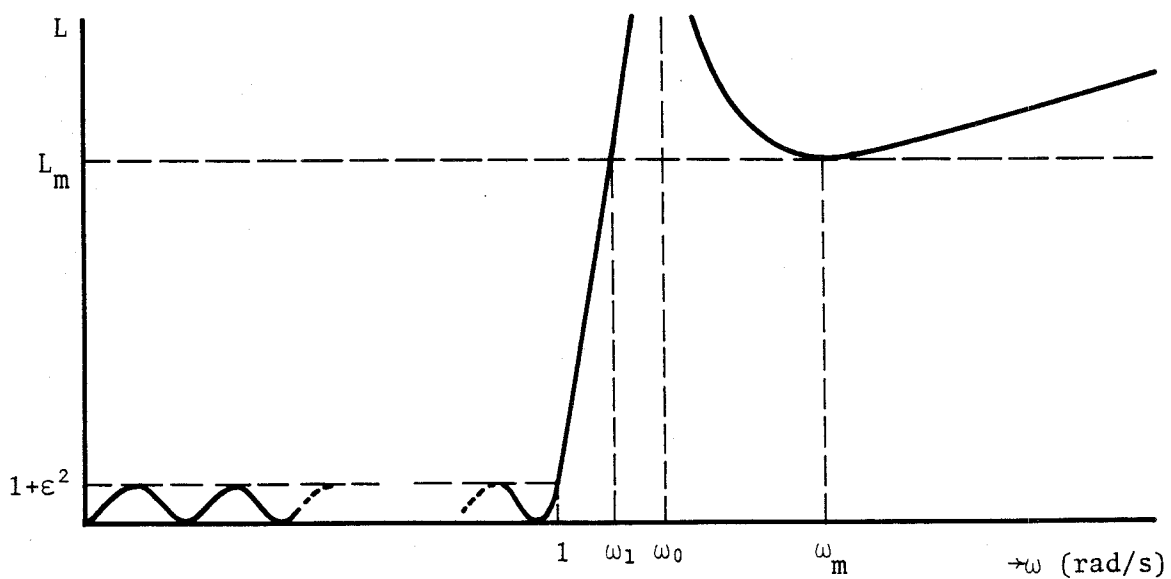


Figure 3. The insertion loss response for a generalised Chebyshev lowpass prototype

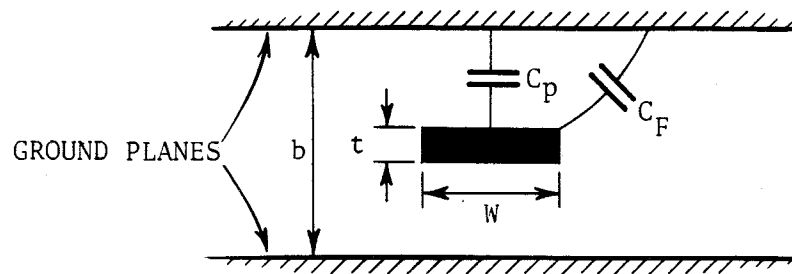


Figure 4. Capacitances present in unshielded stripline

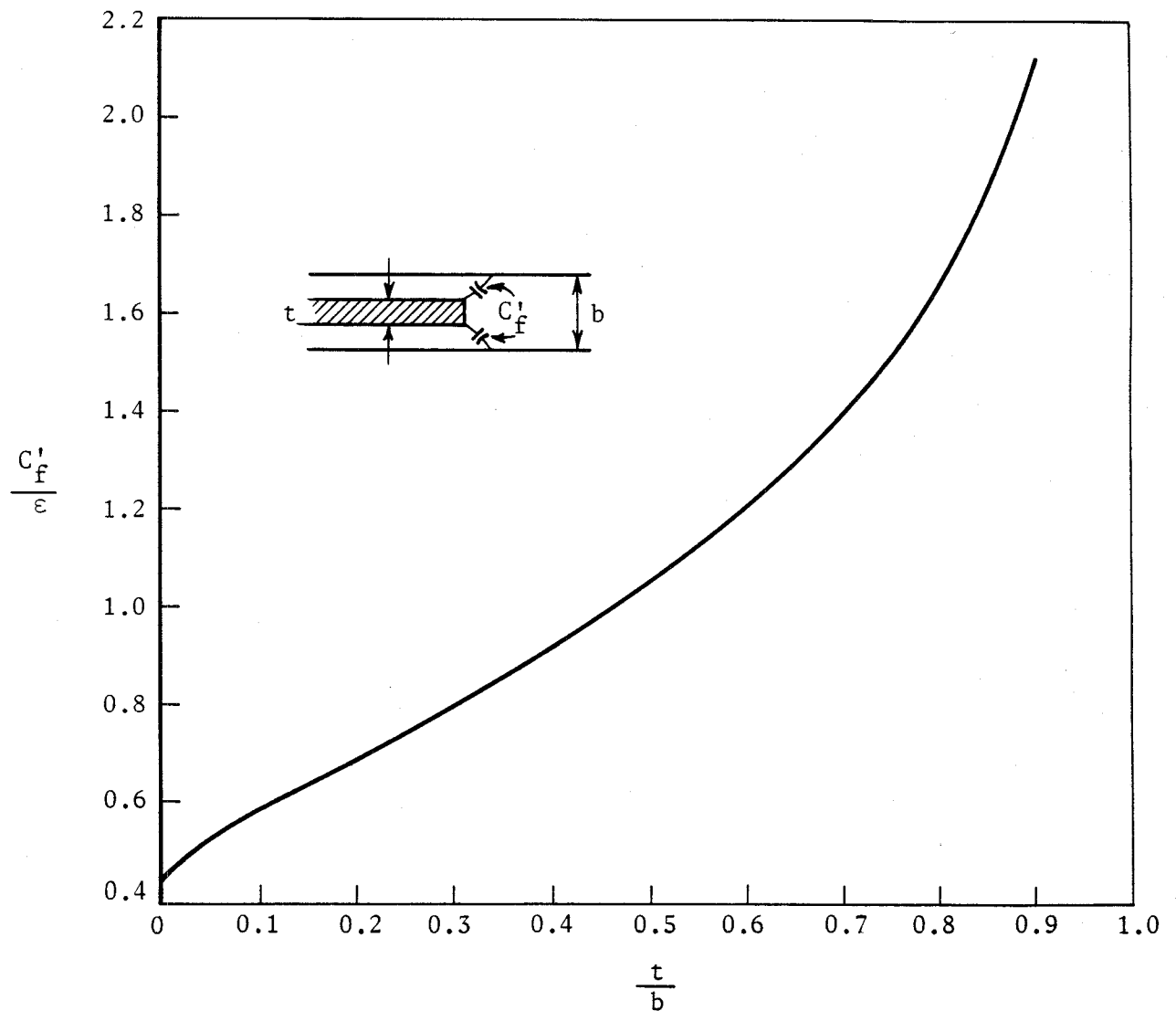
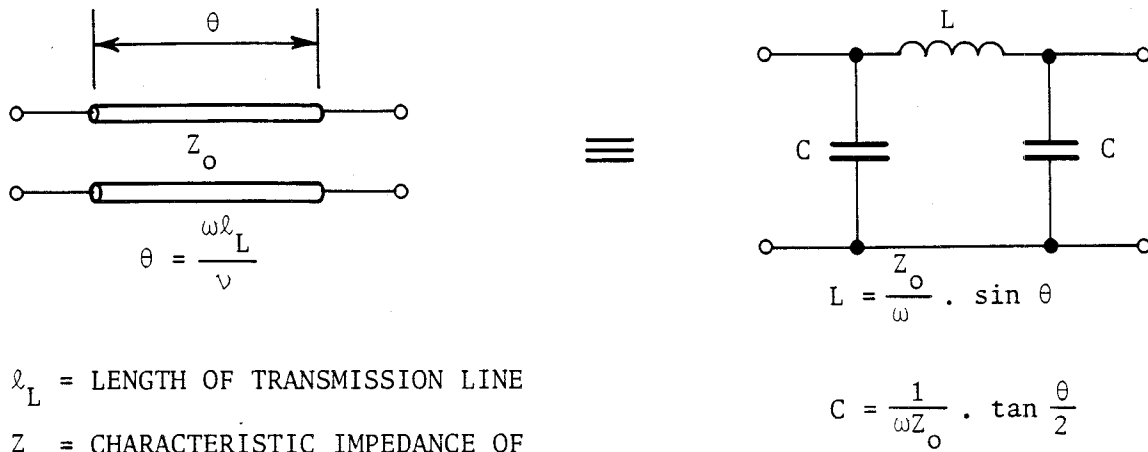


Figure 5. The normalised fringing capacitance for an isolated rectangular bar (see reference 4)



$\ell_L$  = LENGTH OF TRANSMISSION LINE  
 $Z_o$  = CHARACTERISTIC IMPEDANCE OF TRANSMISSION LINE

Figure 6. Equivalent circuit for a length of transmission line

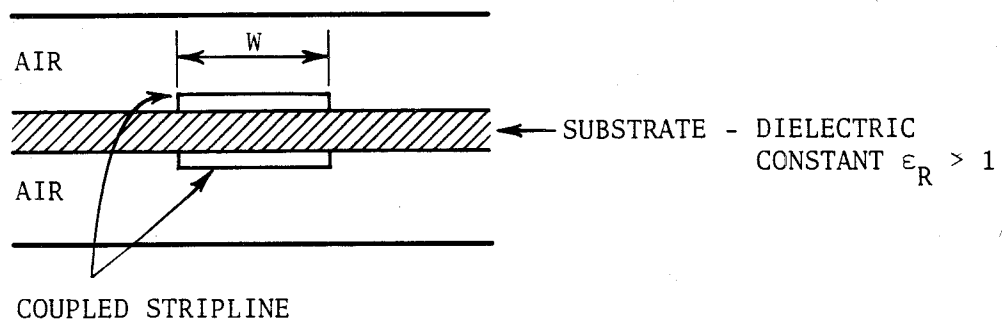


Figure 7. Inhomogeneous broadside - coupled stripline configuration

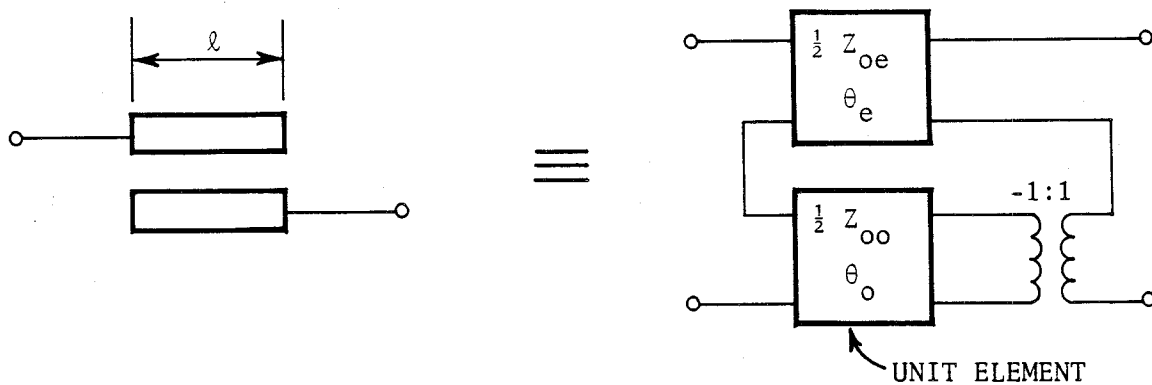


Figure 8. Open-circuit digital section

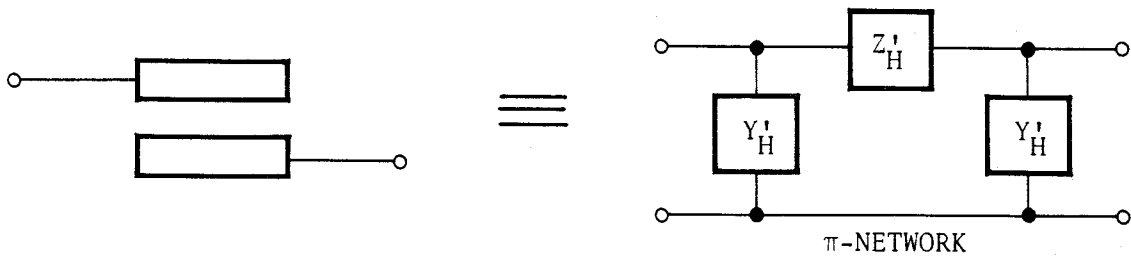


Figure 9. An inhomogeneous coupling section with its equivalent  $\pi$ -network

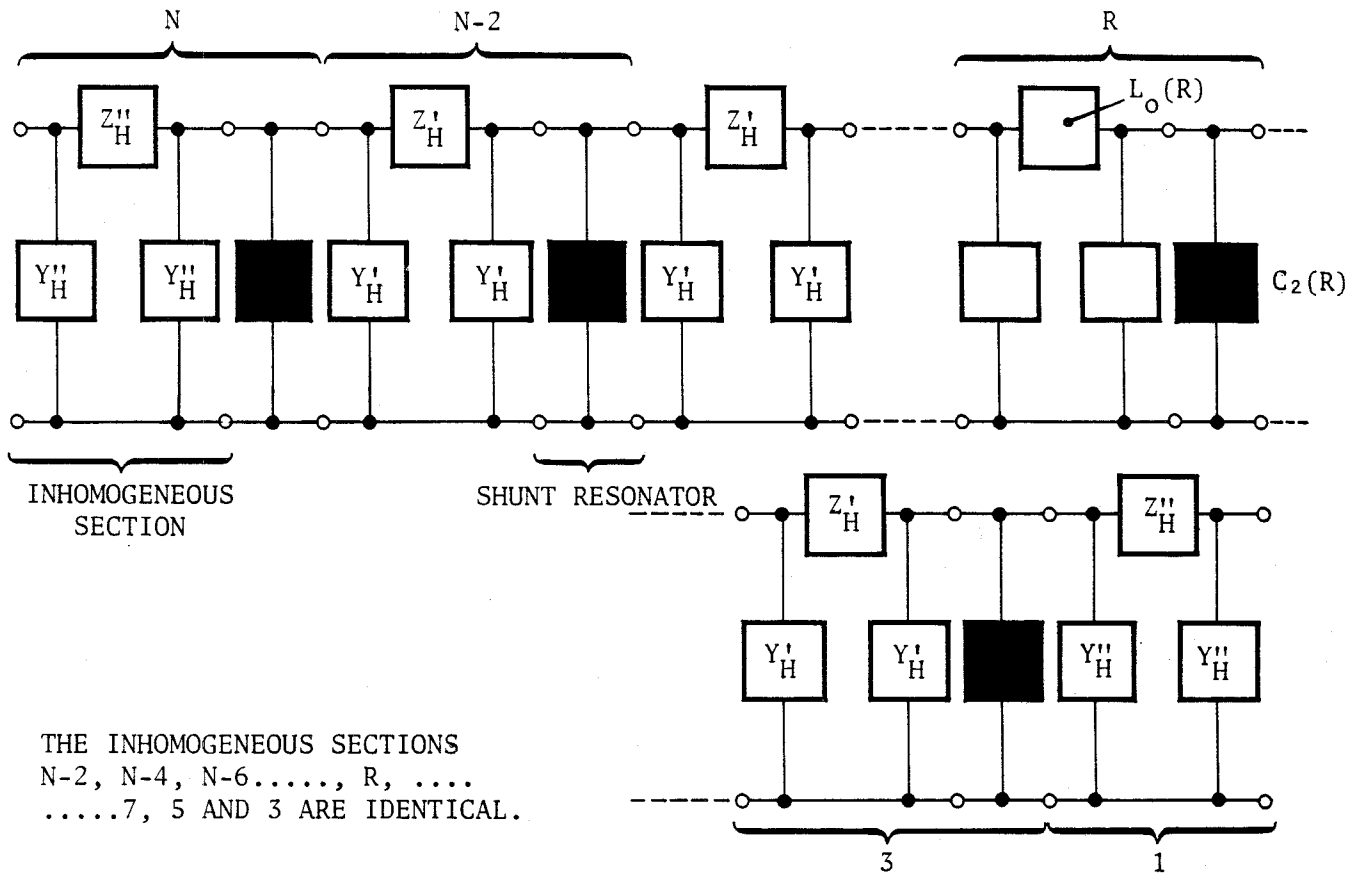


Figure 10. An N-section SSS highpass filter network

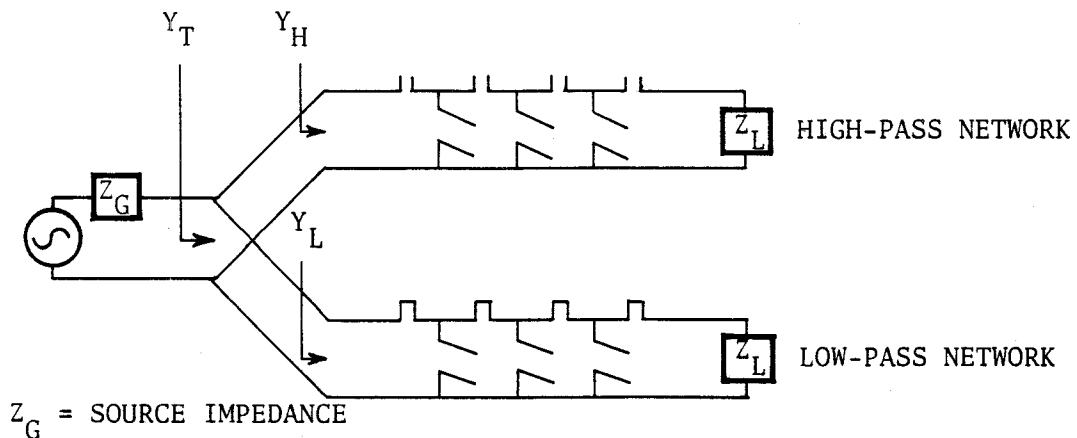


Figure 11. Parallel connected diplexer using high- and lowpass filters

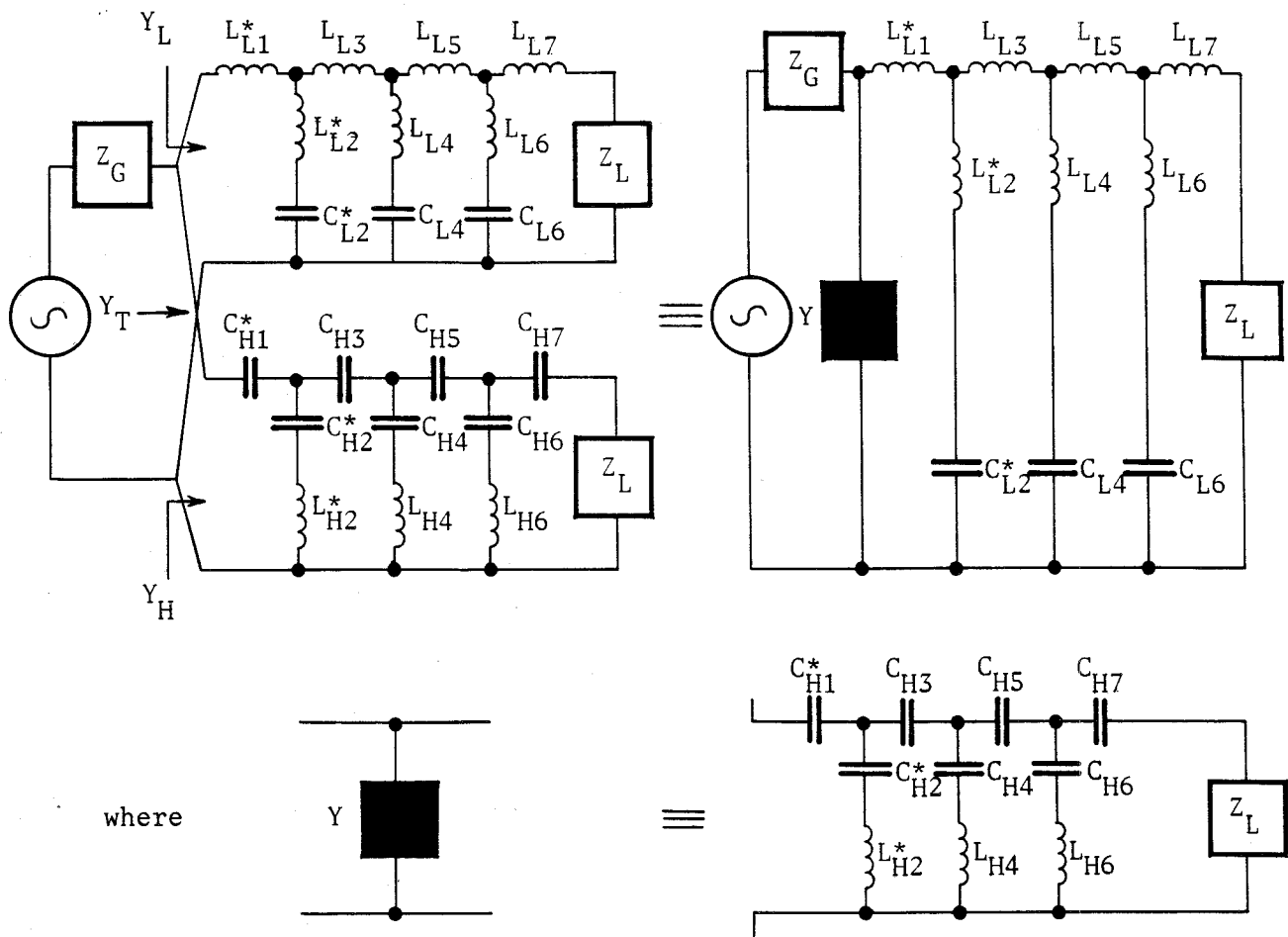


Figure 12. Reduction of a diplexer network into a linear network

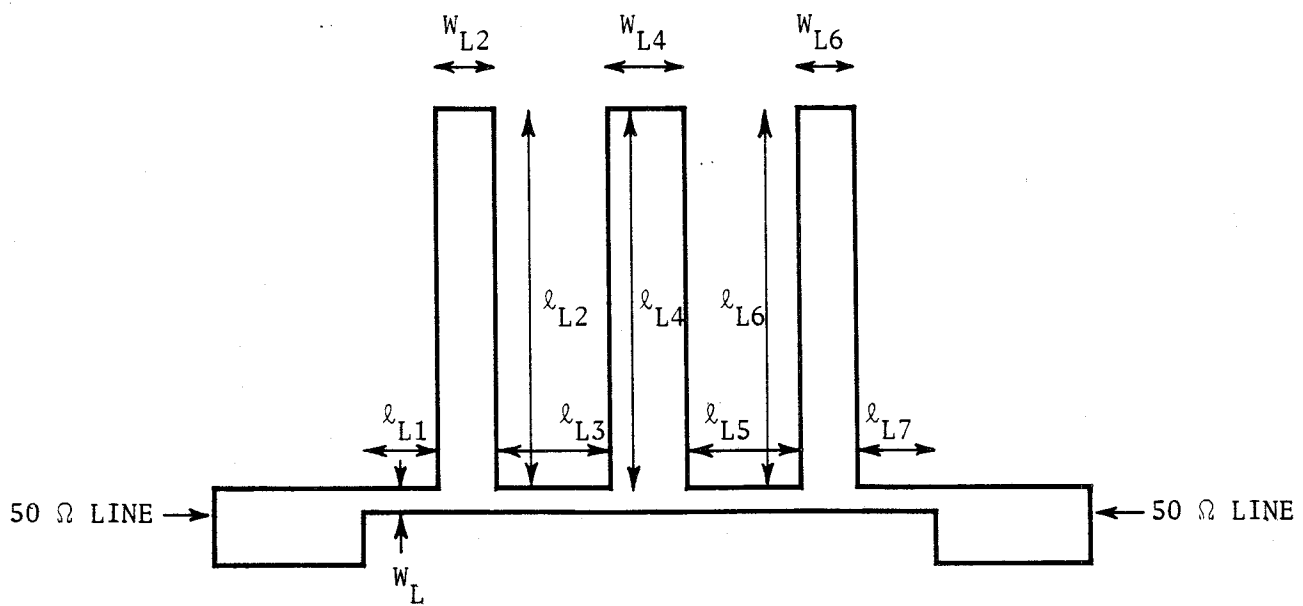


Figure 13. Printed circuit layout for SSS lowpass filter

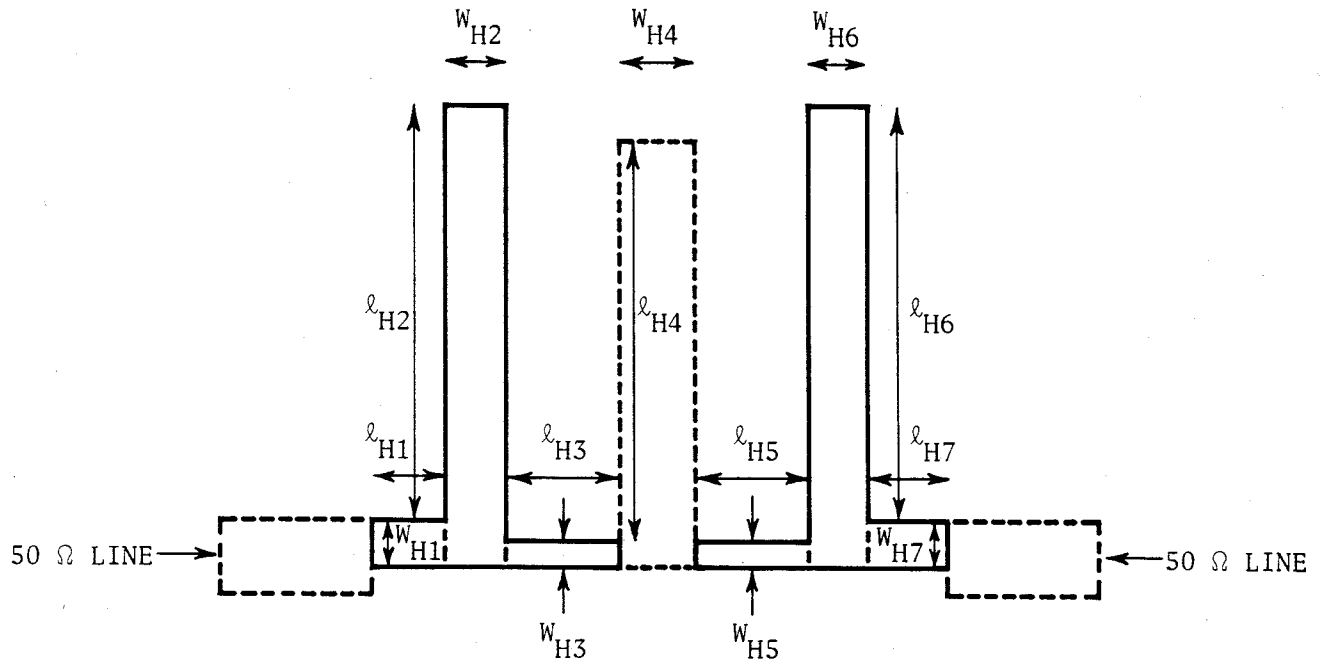


Figure 14. Printed circuit layout for SSS highpass filter. The broken lines outline the remaining circuit on the reverse side of the substrate

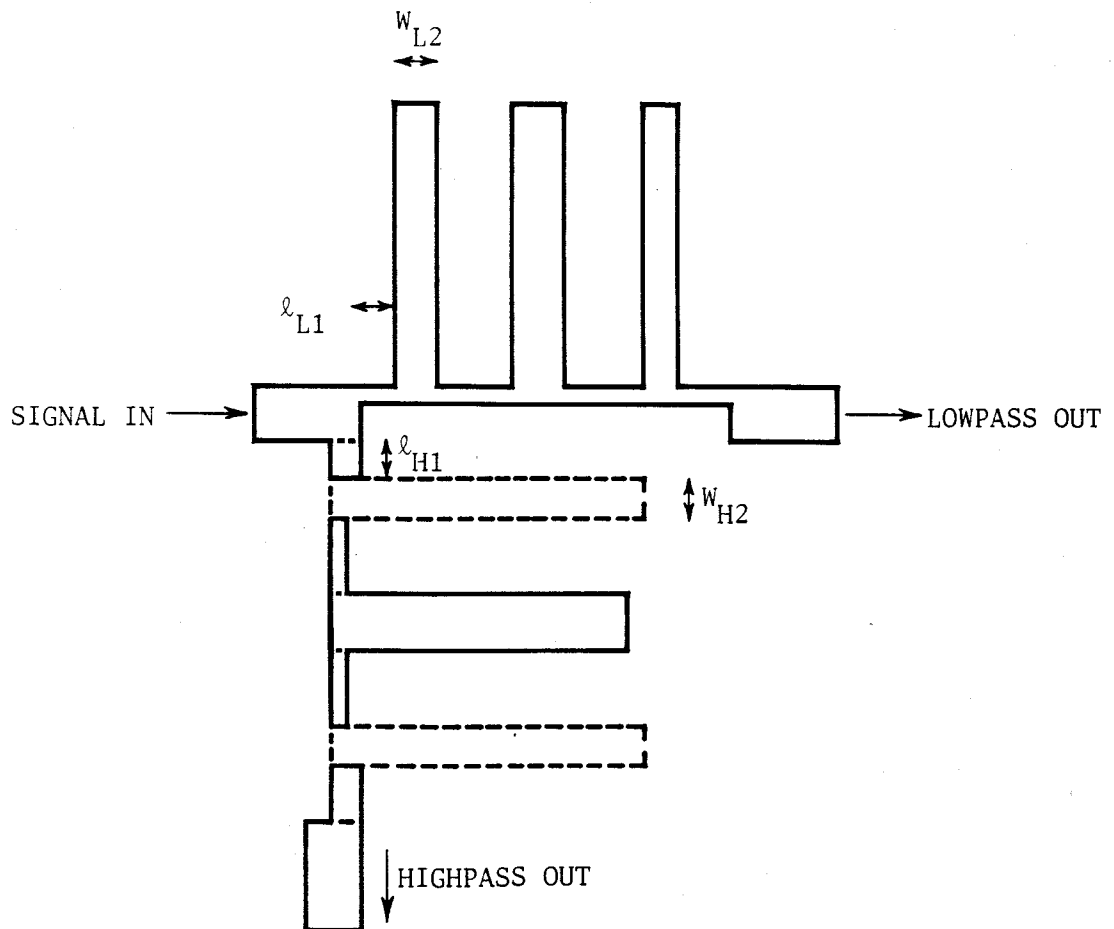


Figure 15. Printed circuit layout for SSS diplexer

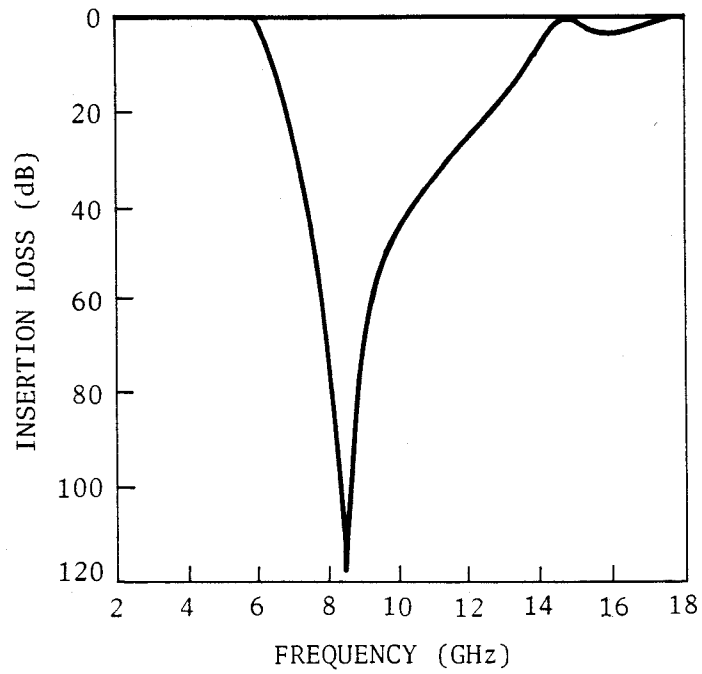


Figure 16. Insertion loss response of 6 GHz SSS lowpass filter

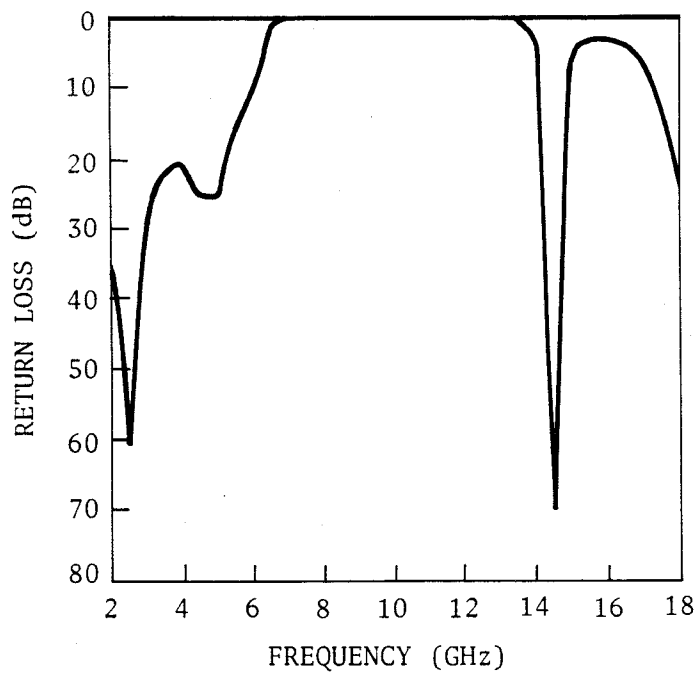


Figure 17. Return loss response of 6 GHz SSS lowpass filter

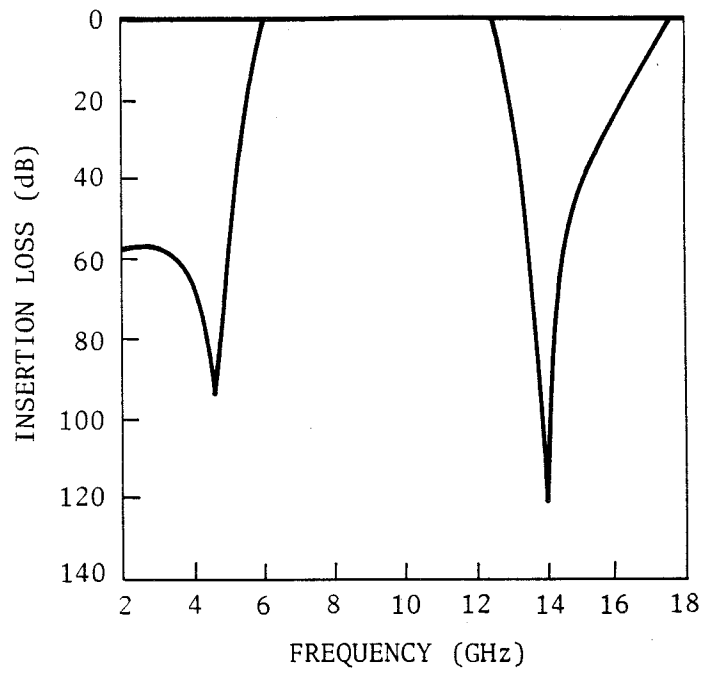


Figure 18. Insertion loss response of 6 GHz SSS highpass filter

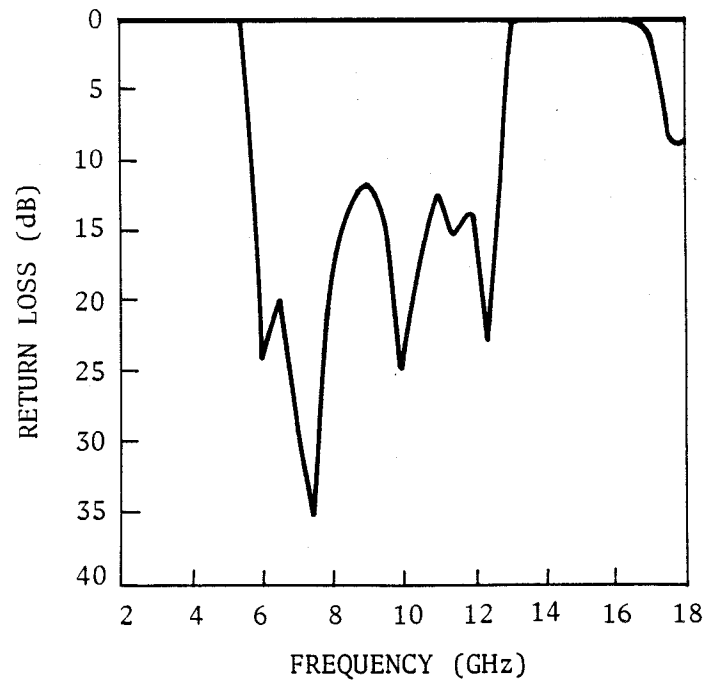


Figure 19. Return loss response of 6 GHz SSS highpass filter



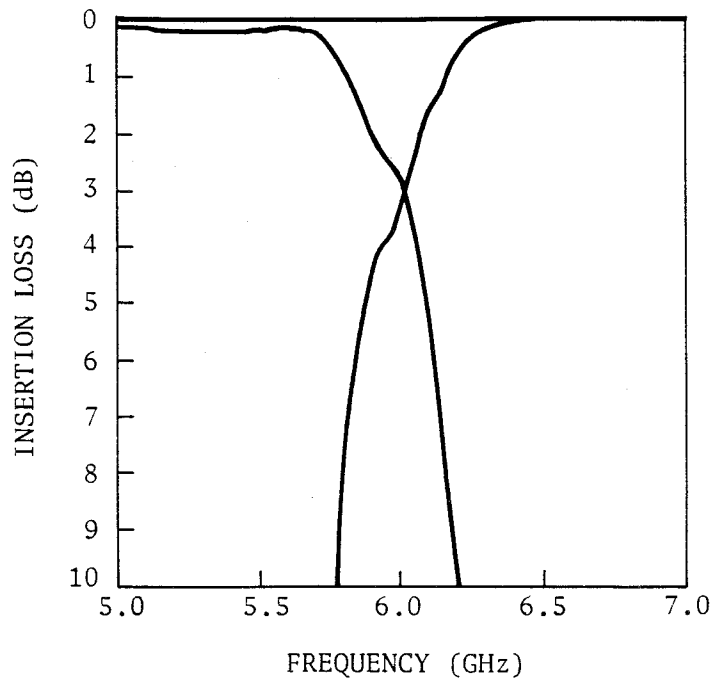


Figure 20. Insertion loss response of a 6 GHz SSS diplexer

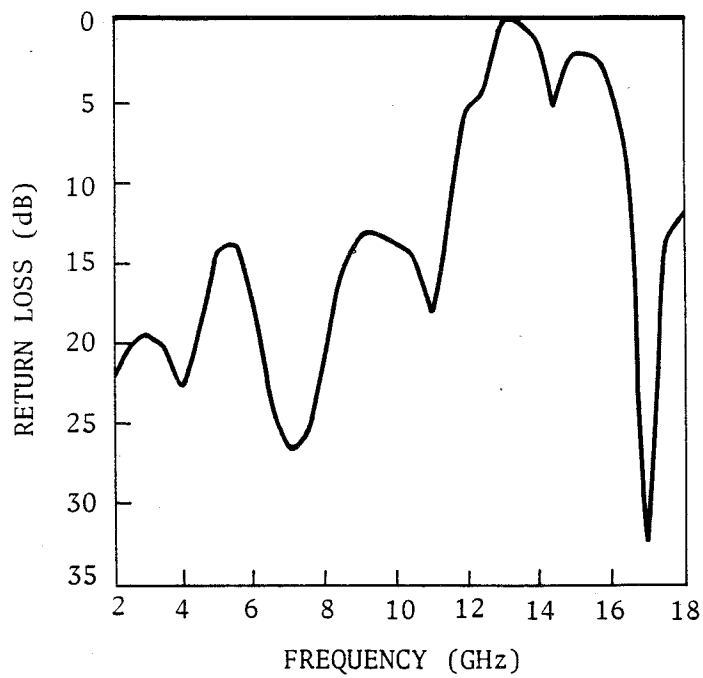


Figure 21. Return loss response of 6 GHz SSS diplexer

# APPENDIX I

## AN ANALYSIS OF INHOMOGENEOUS BROADSIDE-COUPLED STRIPLINES

Inhomogeneous broadside-coupled stripline shown in figure I.1 supports two orthogonal modes of propagation. They are the odd and even modes. For the even mode case, equal currents flow along the two coupling lines. In the odd mode case the currents flowing along both lines are of equal magnitude but opposite direction.

The even and odd mode characteristic impedances, phase velocities and effective dielectric constants are derived from the even and odd mode capacitances. The modal characteristic impedances,  $Z_{oi}$ , and phase velocities  $v_i$  are given by

$$Z_{oi} = \frac{1}{v\sqrt{C_i C_i^a}} \quad (I.1)$$

and

$$v_i = \frac{v}{\sqrt{\epsilon_{ei}}} \quad (I.2)$$

where  $C_i^a$  and  $C_i$  are the capacitances of the coupling structure with air and dielectric respectively. The effective dielectric constants  $\epsilon_{ei}$  are given by

$$\epsilon_{ei} = \frac{C_i}{C_i^a} \quad (I.3)$$

$v$  is the velocity of light in free space. The subscript  $i$  refers to the even or odd mode. For the even mode case  $i=e$  and  $i=o$  for the odd mode.

The variational expression for the modal capacitance  $C_i$  is given by(ref.8)

$$C_i = \frac{\pi \cdot \epsilon_0 \cdot Q^2}{\left[ \int_0^\infty \frac{F^2(\beta) \cdot d\beta}{\left[ \epsilon_{R1} \cdot \left\{ \tanh\left(\frac{\beta s}{2}\right) \right\}^{-m} + \epsilon_{R2} \cdot \coth\left\{\frac{\beta}{2}(b-s)\right\} \right] \cdot \beta} \right]} \quad (I.4)$$

where

$$m = \begin{cases} -1 & \text{for even mode} \\ 1 & \text{for odd mode} \end{cases}$$

$F(\beta)$  is the Fourier transform of the charge distribution  $f(x)$  on the strip conductor.  $Q$  signifies the total charge on the strip conductor and  $\epsilon_0$  is the permittivity of free space. The charge distribution (Coulombs per unit length) on the strip conductor is approximated by

$$f(x) = \begin{cases} 1 + \left| \frac{2 \cdot x}{W} \right|^3, & -W \leq 2x \leq W \\ 0 & \text{elsewhere} \end{cases} \quad (\text{I.5})$$

$F(\beta)$  and  $Q$  are given by

$$F(\beta) = \int_{-\infty}^{\infty} f(x) \cdot e^{j\beta x} dx \quad (\text{I.6})$$

and

$$Q = \int_{-\infty}^{\infty} f(x) dx \quad (\text{I.7})$$

therefore from equations (I.5), (I.6) and (I.7)

$$\frac{Q}{F(\beta)} = \left[ 1.6 \left( \frac{\sin \beta W}{\beta W} \right) + \frac{2.4}{(\beta W)^2} \left\{ \cos \beta W - \frac{2 \sin \beta W}{\beta W} + \frac{\sin^2(\beta W/2)}{(\beta W/2)^2} \right\} \right]^{-1} \quad (\text{I.8})$$

Hence the even and odd mode capacitances may be evaluated by substituting equation (I.8) into equation (I.4).

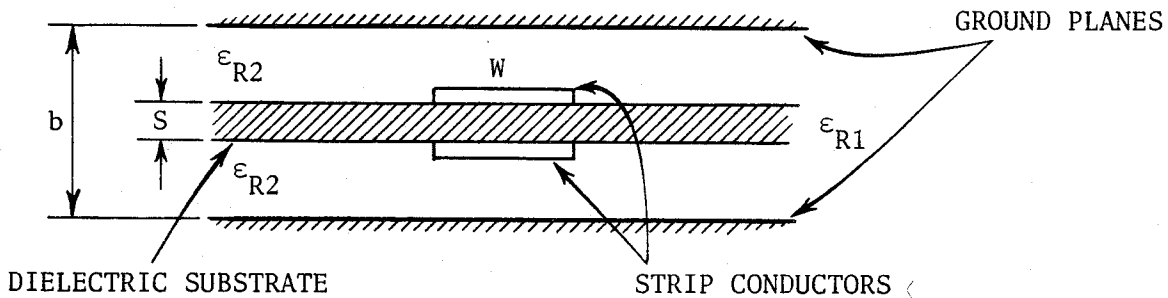
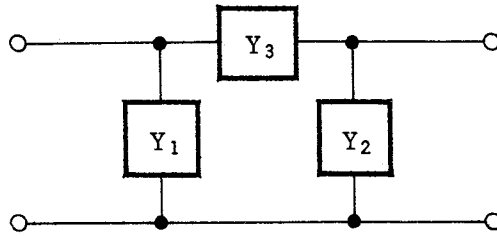


Figure I.1 Broadside-coupled stripline configuration

# APPENDIX II

## GENERAL CIRCUIT PARAMETERS OF A $\pi$ -NETWORK

The ABCD parameters of a  $\pi$ -network having the following form



are given by(ref.9)

$$A = 1 + \frac{Y_2}{Y_3}$$

$$B = \frac{1}{Y_3}$$

$$C = Y_1 + Y_2 + \frac{Y_1 \cdot Y_2}{Y_3}$$

and

$$D = 1 + \frac{Y_1}{Y_3}$$

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### APPENDIX III

#### STRIPLINE DISCONTINUITIES

Common discontinuities which occur in SSS filter realisation are;

1. Open end
2. Step in width, and
3. T-junctions

##### III.1 Open end

The equivalent circuit of an open ended stripline is represented by an excess capacitance. This excess capacitance is equivalent to a length of transmission line  $\Delta\ell$  as shown in figure III.1. The effective open end circuit will be located a distance  $\Delta\ell$  from the physical open end.

$\Delta\ell$  is given by(ref.10)

$$\Delta\ell = \frac{1}{\beta} \cdot \tan^{-1} \left[ \frac{\delta+2W}{4\delta+2W} \cdot \tan(\beta\delta) \right] \quad (\text{III.1})$$

where

$$\delta = \frac{b \cdot \ln 2}{\pi}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{\lambda_o}{\sqrt{\epsilon_R}} \quad (\lambda \cong \lambda_o \text{ for suspended substrate stripline})$$

and  $b$  = ground plane spacing.

Hence equation (III.1) is re-expressed as

$$\Delta\ell = \frac{\lambda_o}{2\pi} \cdot \tan^{-1} \left[ \frac{b \ln 2 + 2\pi W}{4b \ln 2 + 2\pi W} \cdot \tan \frac{2b \ln 2}{\lambda_o} \right] \quad (\text{III.2})$$

The open-circuit capacitance  $C_{oc}$  is related to  $\Delta\ell$  by

$$C_{oc} = \frac{\beta \Delta\ell}{\omega Z_o} \quad (\text{III.3})$$

where  $\omega$  = angular frequency

and  $Z_0$  = characteristic impedance of the stripline.

### III.2 Step in width

This type of discontinuity is formed by the junction of two lines having different widths. The equivalent circuit for a step in width discontinuity is shown in figure III.2. The lengths  $\ell_1$  and  $\ell_2$  (where  $\ell_2 = -\ell_1$ ) explain the shift of the effective junction plane towards the line of smaller width.

The inductive reactance  $X$  is given by (ref.10)

$$\frac{X}{Z_1} = \frac{2 D_1}{\lambda} \cdot \ln \left[ \csc \frac{\pi D_2}{2 D_1} \right] \quad (\text{III.4})$$

If

$$\frac{X}{Z_1} = \frac{\omega \ell_1}{v}$$

then the length  $\ell_1$  is described by the expression

$$\ell_1 = \frac{D_1}{\pi} \cdot \ln \left[ \csc \frac{\pi D_2}{2 D_1} \right] \quad (\text{III.5})$$

where

$$D = \begin{cases} b \cdot \frac{K(k)}{K(k')} + \frac{t}{\pi} \left[ 1 - \ln \left( \frac{2t}{b} \right) \right] & \text{for } \frac{W}{b} \leq 0.5 \\ W + \frac{2b}{\pi} \cdot \ln 2 + \frac{t}{\pi} \left[ 1 - \ln \left( \frac{2t}{b} \right) \right] & \text{for } \frac{W}{b} > 0.5 \end{cases}$$

and

$$\frac{K(k)}{K(k')} = \begin{cases} \frac{1}{\pi} \ln \left( 2 \cdot \frac{1+\sqrt{k'}}{1-\sqrt{k'}} \right) & \text{for } 0 \leq k \leq 0.7 \\ \frac{1}{\pi} \ln \left( 2 \cdot \frac{1+\sqrt{k}}{1-\sqrt{k}} \right) & \text{for } 0.7 < k \leq 1 \end{cases}$$

for

$$k = \tanh\left(\frac{\pi W}{2b}\right) \quad \text{and} \quad k' = \sqrt{1-k^2}$$

### III.3 T-junction

The equivalent circuit for a T-junction is illustrated in figure III.3. The transformer is used as a circuit element to model the discontinuity.

The parameters  $d$  and  $d'$  are the respective reference plane shifts in the main arm and stub arm of the T-junction.

The equivalent strip widths  $D_1$  and  $D_2$  can be determined from(ref.10)

$$D = \begin{cases} b \cdot \frac{K(k)}{K(k')} + \frac{t}{\pi} \cdot \left[ 1 - \ln\left(\frac{2t}{b}\right) \right] & \text{for } \frac{W}{b} \leq 0.5 \\ W + \frac{2b}{\pi} \cdot \ln 2 + \frac{t}{\pi} \cdot \left[ 1 - \ln\left(\frac{2t}{b}\right) \right] & \text{for } \frac{W}{b} > 0.5 \end{cases} \quad (\text{III.6})$$

where  $t$  = thickness of stripline conductor.

The term  $\frac{K(k)}{K(k')}$  is described in Section III.2.

The parameter  $d$  is given by(ref.11)

$$d = \frac{\lambda}{2\pi} \cdot \tan^{-1} \left( -0.7 \cdot \frac{X_a}{Z_{01}} \right) \quad (\text{III.7})$$

where

$$\frac{X_a}{Z_{01}} = - \frac{0.0624 \lambda}{D_1} \cdot \sin^2 \left( \frac{\pi D_2}{\lambda} \right) \quad (\text{III.8})$$

The parameter  $d'$  is given by

$$d' = \frac{\lambda}{2\pi} \cdot \tan^{-1} \left[ \frac{\lambda^2}{\pi^2 D_1 D_2} \cdot \sin^2 \left( \frac{\pi D_2}{\lambda} \right) \cdot \frac{X_b}{Z_{01}} \right] \quad (\text{III.9})$$



where

$$\frac{X_b}{Z_{01}} = \begin{cases} -\frac{X_a}{2Z_{01}} + \frac{2\pi^2 D_1^2 D_2}{\lambda^3} \cdot \frac{1}{\sin^2\left(\frac{\pi D_2}{\lambda}\right)} \cdot \left[ \frac{1}{2} \cdot \left(\frac{D_1}{\lambda}\right)^2 \cdot \left[ \cos^4\left(\frac{\pi D_2}{2D_1}\right) + 3 \right] + \ln \csc\left(\frac{\pi D_2}{2D_1}\right) \right. \\ \left. + \frac{\pi D_2}{6D_1} + \ln 2 \right] & \text{for } \frac{D_2}{D_1} < 0.5 \\ \\ -\frac{X_a}{2Z_{01}} + \frac{2\pi^2 D_1^2 D_2}{\lambda^3} \cdot \frac{1}{\sin^2\left(\frac{\pi D_2}{\lambda}\right)} \left[ \ln\left(\frac{1.43 D_1}{D_2}\right) + 2\left(\frac{D_1}{\lambda}\right)^2 \right] & \text{for } \frac{D_2}{D_1} > 0.5 \end{cases} \quad (\text{III.10})$$

The transformer turns ratio,  $n$ , for the equivalent circuit is given by

$$n = \frac{\lambda}{\pi D_2} \cdot \frac{\sin(\pi D_2/\lambda) \cdot \cos(2\pi d'/\lambda)}{\cos(2\pi d/\lambda)} \quad (\text{III.11})$$

Finally, the shunt susceptance,  $B$ , for the equivalent circuit is given by (ref.11)

$$\begin{aligned} \frac{B}{Y_0} &= 2 \cdot \tan\left(\frac{2\pi d}{\lambda}\right) - n^2 \frac{Y_{02}}{Y_{01}} \cdot \tan\left(\frac{2\pi d'}{\lambda}\right) \\ &= -1.4 \frac{X_a}{Z_{01}} - \frac{n^2 \lambda^2}{\pi^2 \cdot D_1^2} \cdot \sin^2\left(\frac{\pi D_2}{\lambda}\right) \cdot \frac{X_b}{Z_{01}} \end{aligned} \quad (\text{III.12})$$

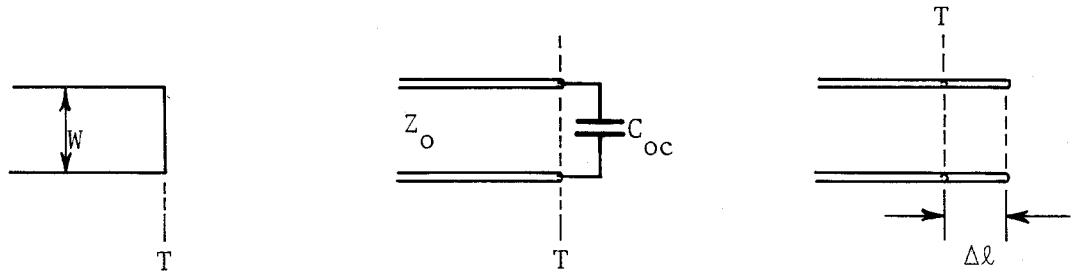


Figure III.1 An open end discontinuity in stripline with its equivalent circuit

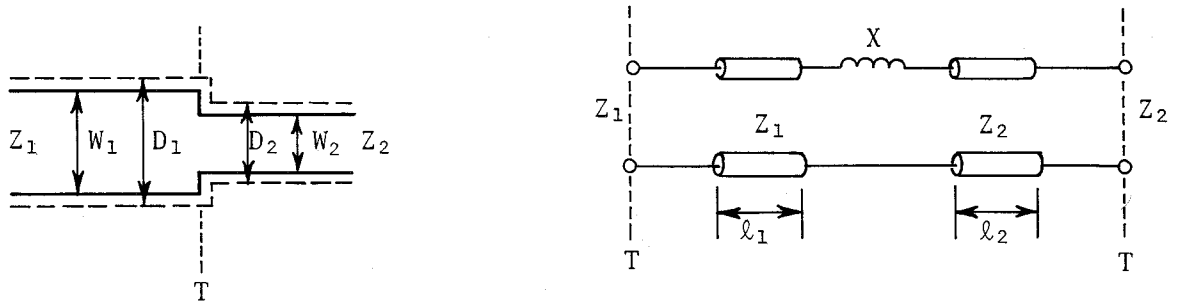


Figure III.2 A step in width discontinuity in stripline with its equivalent circuit

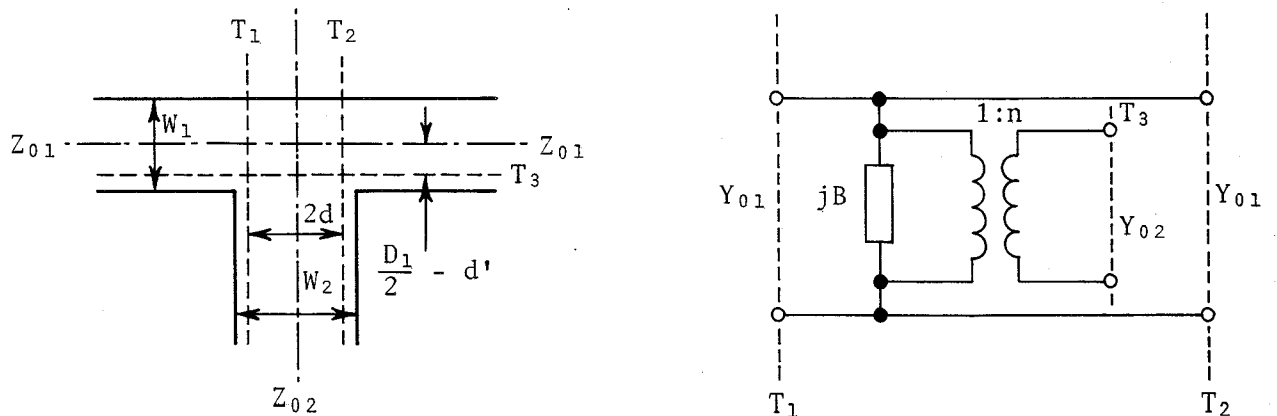


Figure III.3 A T-junction in stripline with its equivalent circuit

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## 16 SUMMARY OR ABSTRACT:

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A method for designing suspended substrate stripline (SSS) lowpass and highpass microwave filters is presented. Such filters can be realised by using a generalised Chebyshev lowpass prototype. An extension of this work is the development of SSS diplexers and multiplexers. The procedure for the design of a SSS contiguous diplexer is given.

SSS filters are compact and light-weight. Furthermore they are not as lossy as microstrip and more conventional stripline microwave filters.